# Nonlinear coupling in the LHC

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# Content

- The observations
- Possible sources
- Simulations
- Analytical formula

- Measurements from June 2012.
- Injection optics, using aperture kicker.
- MO's were powered.
- Amplitude dependent coupling $\mathrm{dQ}_{\mathrm{min}}$ was observed.
- $dQ_{min}$ was around 0.03

## Single particle emittance (action)

$$\epsilon_{x,y} \equiv 2J_{x,y}$$

## Action measurement

$$2J_{x,y} = \frac{1}{N_{\text{bpm}}} \sum_{i}^{N_{\text{bpm}}} \frac{(0.5A_{x,y})^2}{\beta_{x,y}}$$

$N_{\text{bpm}}$ is the number of BPM's, $A$ is the peak-to-peak amplitude.

# Observations

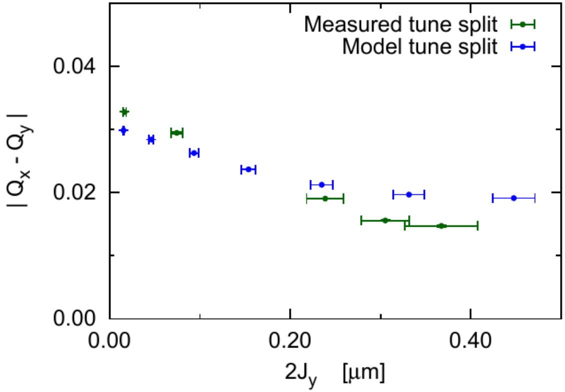



FIG. 7: Modelled and measured variation in the tune-split with vertical kick amplitude at nominal injection optics.

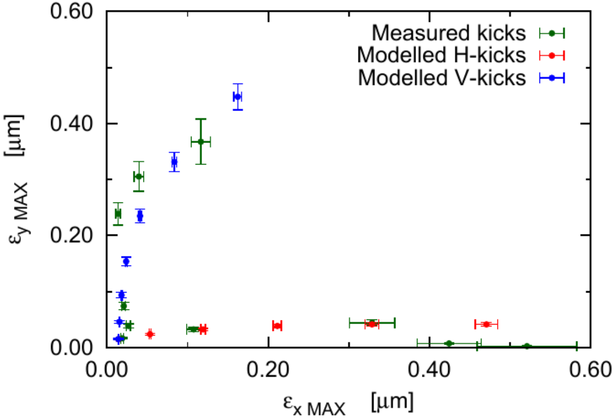



FIG. 9: Comparison of the modelled and measured $\epsilon_{MAX}$ of kicks in the horizontal or vertical planes at nominal injection optics.

# Observations

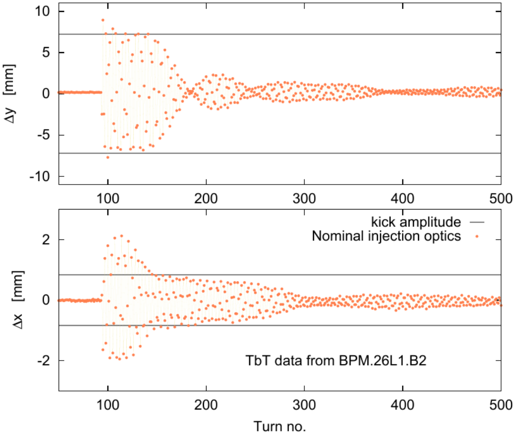


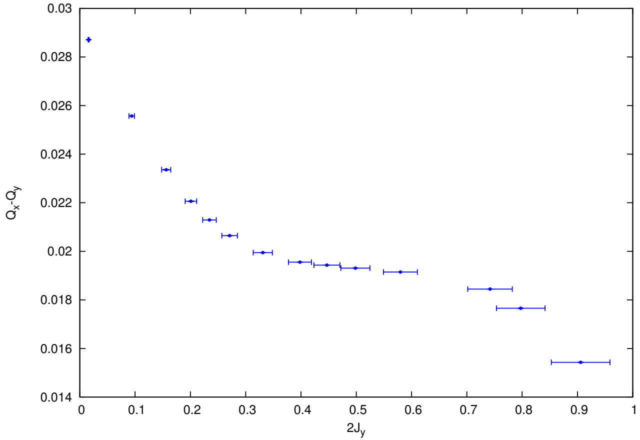

FIG. 10: Measured TbT data from a large amplitude vertical kick at nominal injection optics.

# Possible Sources

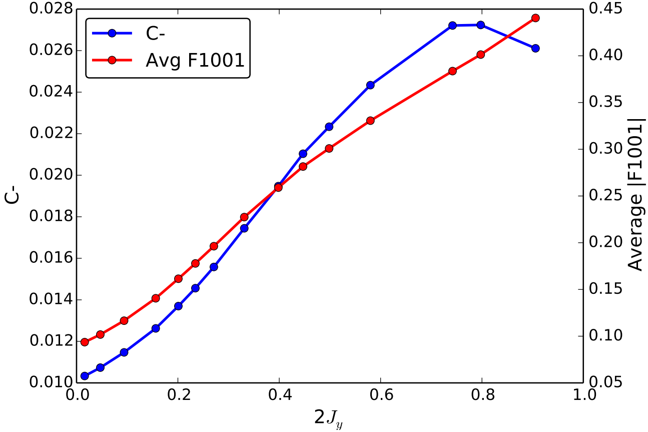
- Skew octupoles
- Normal octupoles + linear coupling (effectively skew oct.)
- Sextupoles + skew sextupoles
- Sextupoles + coupling

- Only simulate kicks in vertical plane (for now).
- Track 1 particle in ptc_track, with starting x,y in IP3, for each kick.
- Always start with x = 0.5 mm (lowest kick in y).
- Look at $Q_x - Q_y$ as a function of action from getkick.out, GetLLM with all files in one run.
- Look at $dQ_{min}$ from getcouple.out from independent runs of GetLLM.

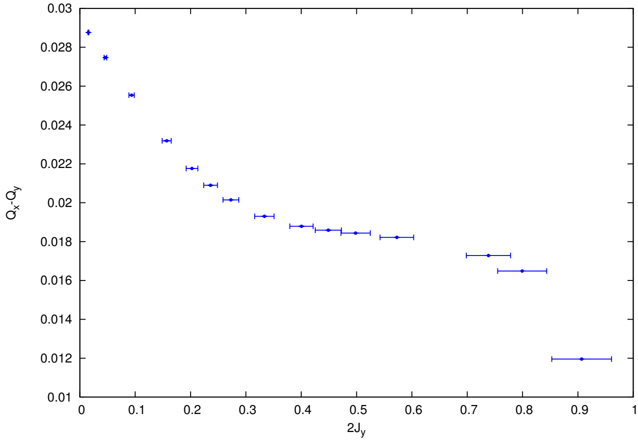
# Base Machine with Known Misalignments/Errors



# Base Machine with Known Misalignments/Errors



# Example - Removing A4 errors in dipoles



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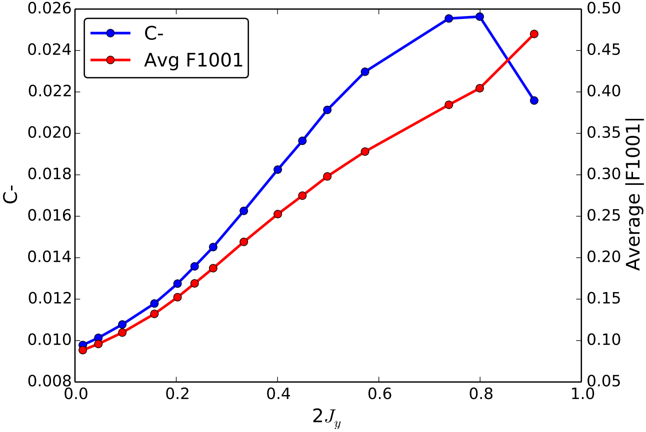
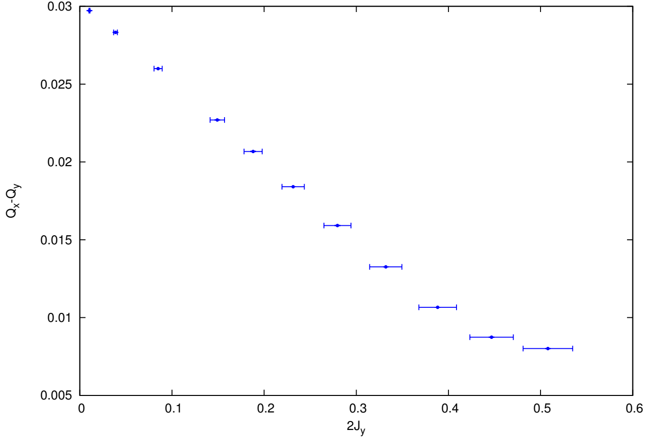
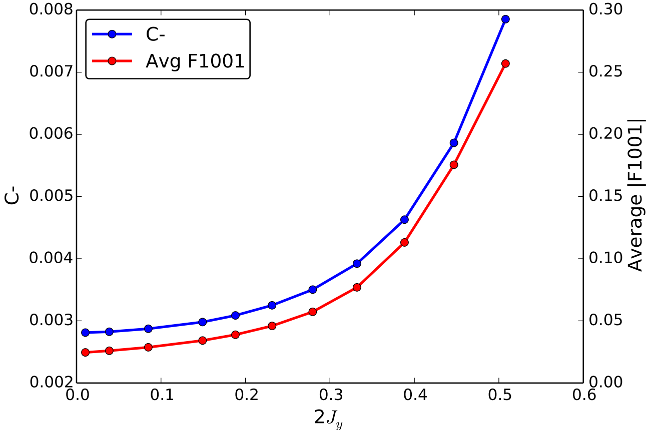


# Removing all sources of linear coupling



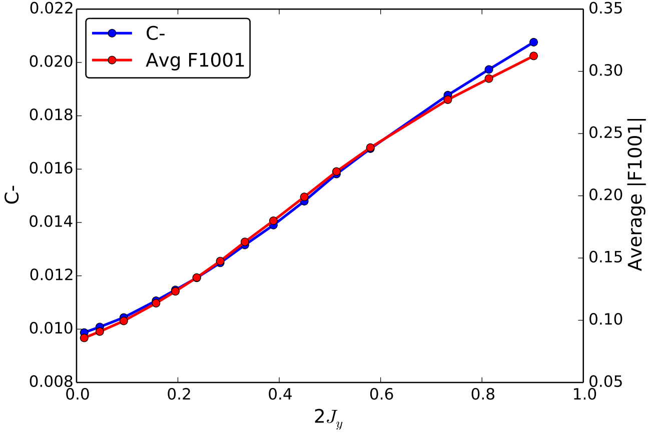

vertical misalignments and roll of sextupoles, quadrupoles, and dipoles.

# Removing all sources of linear coupling



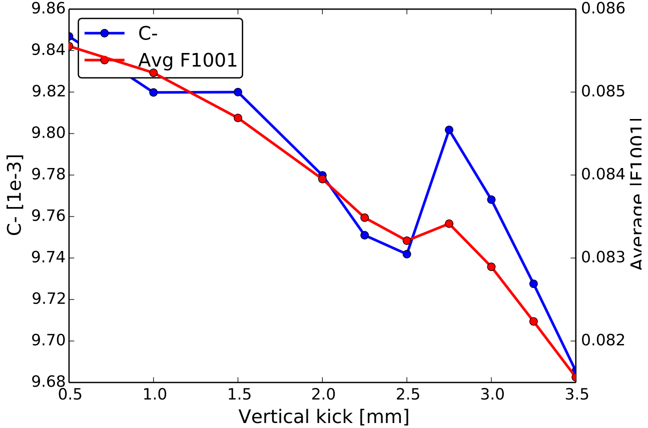
# Ideal Machine w/coupling

Ideal machine, dQmin is matched to 0.01, MO's are powered.



# Ideal Machine w/coupling - No MO's

Ideal machine, dQmin is matched to 0.01, MO's are **not** powered.



**Hamiltonian**

$$H = \sum_{w} \sum_{jklm} h_{w,jklm} e^{i[(j-k)\phi_{w,x}+(l-m)\phi_{w,y}]} h_x^{+j} h_x^{-k} h_y^{+l} h_y^{-m}$$

$$h^+ h^- = 2J$$

$$h_{w,jklm} = -i^{l+m} \frac{K_{w,n-1} + iJ_{w,n-1}}{j!\ k!\ l!\ m!\ 2^n} \beta_{w,x}^{\frac{j+k}{2}} \beta_{w,y}^{\frac{l+m}{2}}$$

**Hamiltonian - Linear + NL Coupled Motion**

$$H = \sum_{w} \left(h_{w,1001} + h_{w,2101} 2J_x + h_{w,1012} 2J_y\right) e^{i[\phi_{w,x}-\phi_{w,y}]} h_x^+ h_y^- .$$

## Normal Octupole Terms

$$x^4 : h_{4000}, h_{3100}, h_{2200} h_{1300}, h_{0400}$$

$$y^4 : h_{0040}, h_{0031}, h_{0022} h_{0013}, h_{0004}$$

$$x^2 y^2 : h_{2020}, h_{2011}, h_{2002}, h_{1120}, h_{1111}, h_{1102}, h_{0220}, h_{0211}, h_{0202}$$

## Coupled Basis

$$\begin{aligned}
\xi_x^+ &= h_x^+ + 2if_{0110}h_y^+ + 2if_{0101}h_y^- \\
\xi_x^- &= h_x^- - 2if_{1010}h_y^+ - 2if_{1001}h_y^- \\
\xi_y^+ &= h_y^+ + 2if_{1001}h_x^+ + 2if_{0101}h_x^- \\
\xi_y^- &= h_y^- - 2if_{1010}h_x^+ - 2if_{0110}h_x^-
\end{aligned}$$

## The Additional Hamiltonian, to first order

$$\Delta H = \sum_j \frac{\partial H}{\partial h_j} \times (\xi_j - h_j)$$

$$H_0 \equiv h_{w,jklm} e^{i[(j-k)\phi_{w,x} + (l-m)\phi_{w,y}]} h_x^{+j} h_x^{-k} h_y^{+l} h_y^{-m}$$

$$
\begin{aligned}
&2iH_0[\\
&\quad + \frac{j}{h_x^+}\left(h_y^+ f_{w,0110} e^{i(-\phi_{w,x}+\phi_{w,y})} + h_y^- f_{w,0101} e^{i(-\phi_{w,x}-\phi_{w,y})}\right)\\
&\quad - \frac{k}{h_x^-}\left(h_y^+ f_{w,1010} e^{i(\phi_{w,x}+\phi_{w,y})} + h_y^- f_{w,1001} e^{i(\phi_{w,x}-\phi_{w,y})}\right)\\
&\quad + \frac{l}{h_y^+}\left(h_x^+ f_{w,1001} e^{i(+\phi_{w,x}+\phi_{w,y})} + h_x^- f_{w,0101} e^{i(-\phi_{w,x}+\phi_{w,y})}\right)\\
&\quad - \frac{m}{h_y^-}\left(h_x^+ f_{w,1010} e^{i(+\phi_{w,x}-\phi_{w,y})} + h_x^- f_{w,0110} e^{i(-\phi_{w,x}-\phi_{w,y})}\right)\\
&]
\end{aligned}
$$

$$2101 terms: 6ih_{w,3100}f_{w,0101}, -4ih_{w,2002}f_{w,0110},$$
$$-4ih_{w,1102}f_{w,1010}, 2ih_{w,2011}f_{w,0101},$$
$$-4ih_{w,2200}f_{w,1001}, 2ih_{w,1111}f_{w,1001}$$

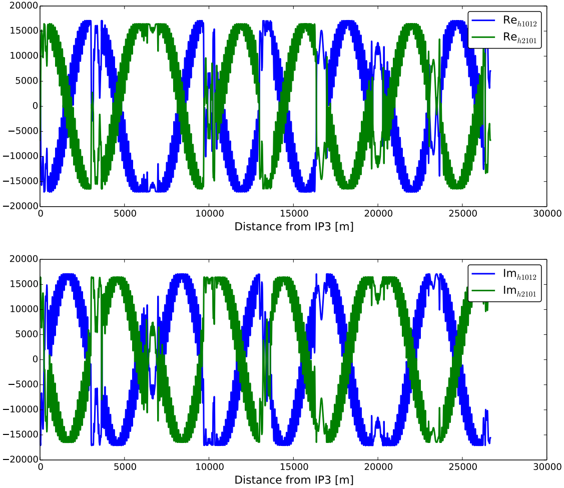
$$1012 terms: -2ih_{w,1102}f_{w,1010}, 4ih_{w,2011}f_{w,0101},$$
$$4ih_{w,2002}f_{w,0110}, -6ih_{w,0013}f_{w,1010}$$
$$4ih_{w,0022}f_{w,1001}, -2ih_{w,1111}f_{w,1001}$$

Inserting $h_{w,jklm}$, we get

$$k_{w,2101} = \frac{iK_{w,3}\beta_{w,x}}{16}[\beta_{w,x}(-f_{w,0101} + f_{w,1001}) + \beta_{w,y}(-f_{w,0110} - 2f_{w,1010} + f_{w,0101} + 2f_{w,1001})$$

$$h_{w,1012} = \frac{iK_{w,3}\beta_{w,y}}{16}[\beta_{w,x}(f_{w,0110} + 2f_{w,0101} - f_{w,1010} - 2f_{w,1001}) + \beta_{w,y}(f_{w,1010} - f_{w,1001})]$$

# Implementing in metaclass



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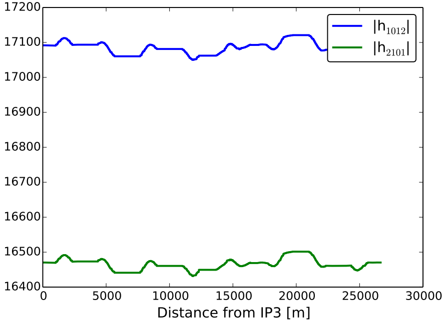


PTC_NORMAL(IP3):

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"HAMA" 1 0 1 2 **28048**

# Next Actions

- Toy model to understand the effect better
- Check analytical formula thoroughly for errors
- Figure out the importance of sextupoles + skew sext./coupling