PS Booster resonance compensation measurements

Meghan McAteer
29 April 2014

Outline

- Review of some basics
  - Resonant behavior in synchrotrons
  - Multipole expansion of magnetic fields
- Hamiltonian formalism for nonlinear optics
- Results of first measurements in the PSB

Part I
Review of some basics

Resonant behavior in synchrotrons

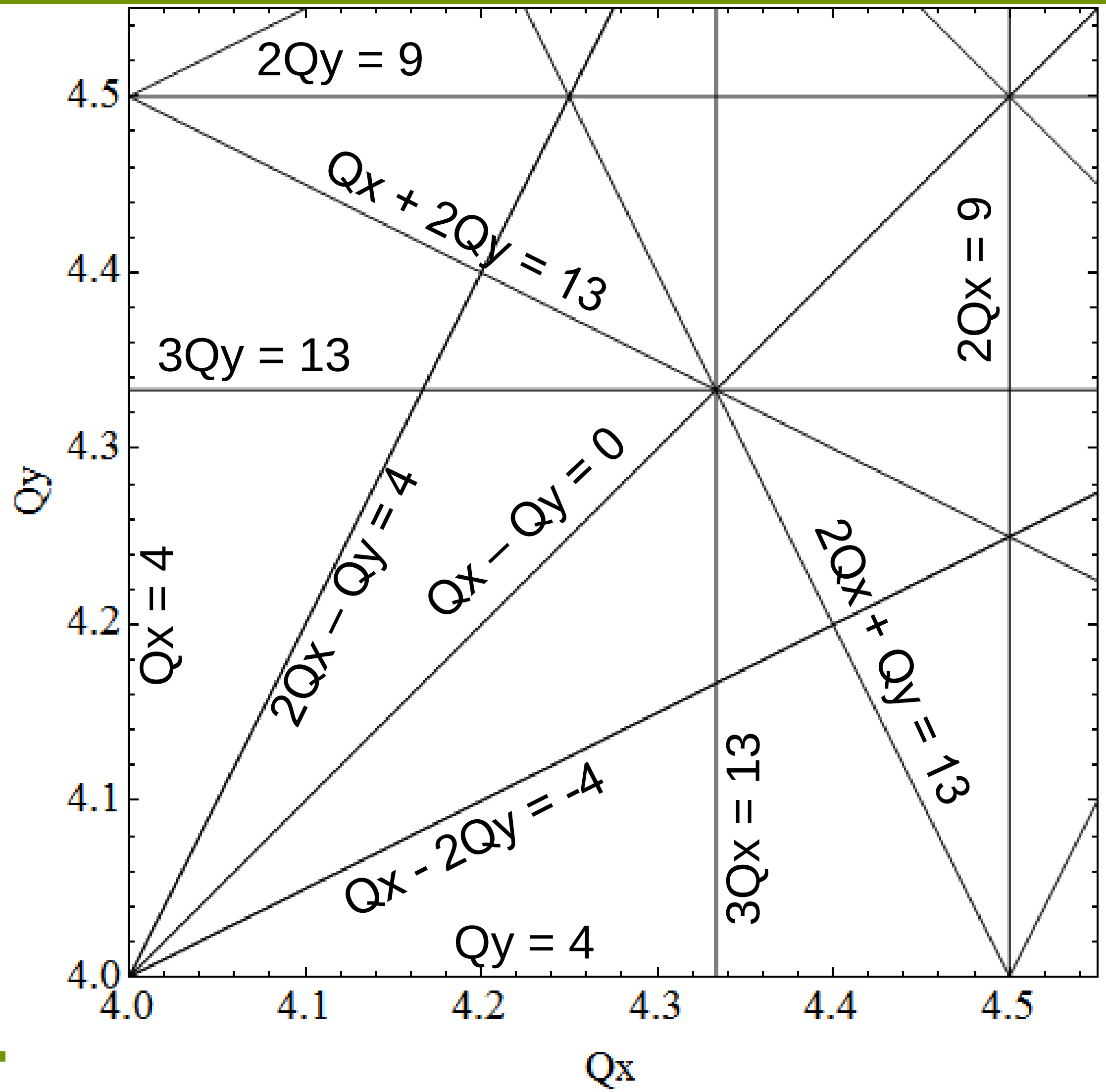
- In general, resonant behavior can occur whenever

$$n_1 Q_x + n_2 Q_y = integer$$

order of resonance:

$$n = n_1 + n_2$$

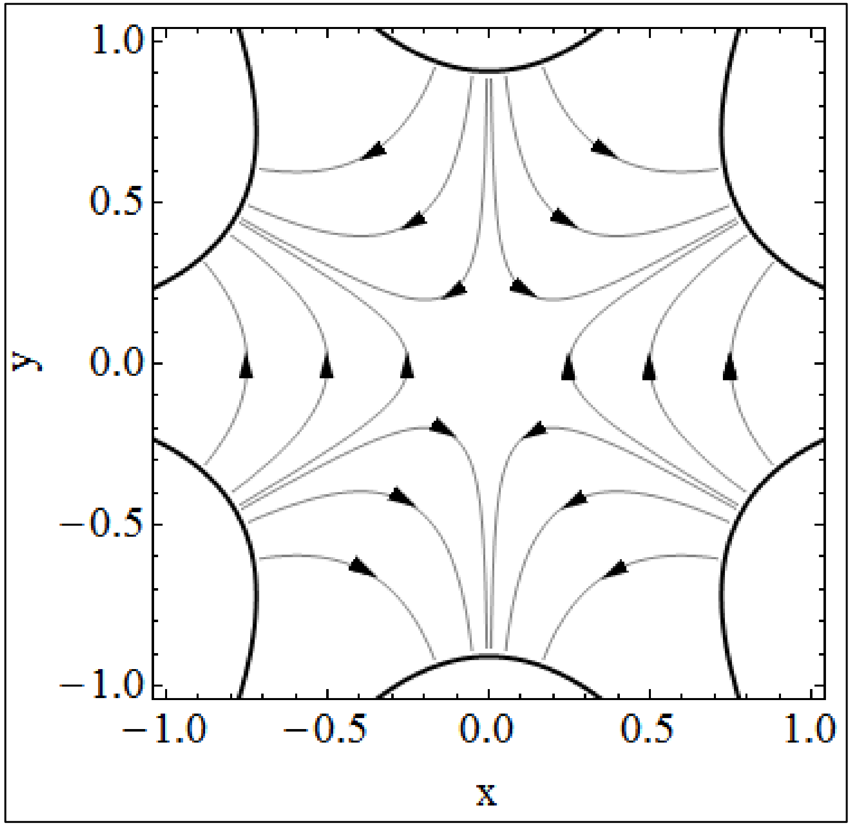
- *nth* order multipole magnet perturbation can excite resonances up to *nth* order



Complex potential of magnetic multipoles

- Multipole magnets can be described by complex magnetic potential:

$$\Psi = \sum_n \frac{1}{n}(B_n + iA_n)(x+iy)^n$$
$$= A_s(x,y) + iV(x,y)$$



- Hamiltonian is proportional real part of ψ
- Example: normal sextupole (n=3, A3=0)

$$A_s(x,y) = Re\left[\frac{1}{3}(B_3 + iA_3)(x+iy)^3\right]$$
$$= \frac{1}{3}B_3\left(x^3 - 3xy^2\right)$$
$$V(x,y) = Im\left[\frac{1}{3}(B_3 + iA_3)(x+iy)^3\right]$$
$$= \frac{1}{3}B_3\left(3x^2y - 3y^3\right)$$

- Vector equipotentials=field lines, scalar equipotential=pole face contour

Part II
Hamiltonian formalism for resonance measurement

The general approach

- Step 1: define a method of mapping nonlinear particle motion in an accelerator (Taylor maps)

- Step 2: define the relationship between map and machine observables (Fourier spectra of transverse beam trajectories)
- Step 3: identify resonance driving terms

- Trajectory through a multipole can be mapped using the Hamiltonian of the multipole magnet:

Taylor maps for nonlinear magnet elements

- Maps for nonlinear lattice elements can't be written in terms of transfer matrices; need new approach (exponential Lie operators)

- Definition of exponential Lie operator $e^{:f:}$ acting on a function g:

$$e^{:f:}g = g + [f,g] + \frac{1}{2}[f,[f,g]] + \ldots$$

$$[f,g] = \frac{\partial f}{\partial \vec{x}}\frac{\partial g}{\partial \vec{p}} - \frac{\partial f}{\partial \vec{p}}\frac{\partial g}{\partial \vec{x}}$$ (Poisson bracket)

Recall Taylor series expansion of exponential function:

$$e^{f} = 1 + \frac{f^{1}}{1!} + \frac{f^{2}}{2!} + \ldots$$

- Particle's coordinates $\vec{X}$ after passage through a multipole can be mapped using the Hamiltonian of the multipole magnet:

$$\vec{X}_f = e^{:-H:}\vec{X}_0$$

Recall from classical mechanics: time evolution of a function g(x,px) from Poisson bracket with Hamiltonian

$$\frac{dg}{dt} = [g,H]$$

Example: Taylor map for a thin-lens normal sextupole kick (1 of 2)

- Hamiltonian for an nth order multipole kick is proportional to the vector potential for the multipole:

$$h = \frac{qL}{p_0} Re\left[\frac{1}{n}(B_n + iA_n)(x+iy)^n\right]$$

L is magnet length,
q is particle charge,
p_0 is momentum

- So for a thin-lens normal sextupole (n=3, A3=0), the Hamiltonian is

$$h = \frac{qLB_3}{3p_0}\left(x^3 - 3xy^2\right)$$

- And the map relating trajectory before and after the sextupole kick is

$$\begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix}_f = e^{:-\frac{qLB_3}{3p_0}\left(x^3-3xy^2\right):} \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix}_0$$

Example: Taylor map for a thin-lens normal sextupole kick (2 of 2)

- Hamiltonian for normal sextupole lens: $$h = \frac{qLB_3}{3p_0}\left(x^3 - 3xy^2\right)$$

- The derivatives of h (for the Poisson bracket $[h, \vec{X}]$) are

$$\frac{\partial h}{\partial x} = -\frac{qLB_3}{p_0}\left(x^2 - 3y^2\right) \qquad \frac{\partial h}{\partial y} = \frac{qLB_3}{p_0}(6xy) \qquad \frac{\partial h}{\partial p_x} = \frac{\partial h}{\partial p_y} = 0 \qquad \frac{\partial \vec{X}}{\partial x} = \frac{\partial \vec{X}}{\partial p_x} = \frac{\partial \vec{X}}{\partial y} = \frac{\partial \vec{X}}{\partial p_y} = 1$$

and so the new coordinates after the sextupole kick are

$$\begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix}_f = \begin{pmatrix} x_0 + \frac{\partial h}{\partial p_x} \\ p_{x0} - \frac{\partial h}{\partial x} \\ y_0 + \frac{\partial h}{\partial p_y} \\ p_{y0} - \frac{\partial h}{\partial y} \end{pmatrix} = \begin{pmatrix} x_0 \\ p_{x0} + \frac{qLB_3}{p_0}\left(x_0{}^2 - y_0{}^2\right) \\ y_0 \\ p_{y0} - \frac{qLB_3}{p_0}(2x_0 y_0) \end{pmatrix}$$

The Hamiltonian for many multipole kicks (1 of 2)

- To first order, the Hamiltonian for all multipole elements in the ring can be expressed as a sum over *i* individual multipole elements:

$$h = \sum_i h_i = \frac{qL_i}{p_0} Re\left[\sum_n \frac{1}{n}\left(B_n(s_i) + iA_n(s_i)\right)\left(x(s_i) + iy(s_i)\right)^n\right]$$

- Insert expression for x and y (solutions to unperturbed equations of motion):

$$x(s_i) = \sqrt{2J_x\beta_x(s_i)}Cos(\varphi_x(s_i) + \varphi_{x0}) = \sqrt{2J_x\beta_x(s_i)}\frac{e^{i(\varphi_x(s_i)+\varphi_{x0})} + e^{-i(\varphi_x(s_i)+\varphi_{x0})}}{2}$$

$$y(s_i) = \sqrt{2J_y\beta_y(s_i)}Cos(\varphi_y(s_i) + \varphi_{y0}) = \sqrt{2J_y\beta_y(s_i)}\frac{e^{i(\varphi_y(s_i)+\varphi_{y0})} + e^{-i(\varphi_y(s_i)+\varphi_{y0})}}{2}$$

The Hamiltonian for many multipole kicks (2 of 2)

- Recall: Multinomial expansion of polynomials

$$(a+b+c+d)^n \equiv \sum_{j+k+l+m\leq n} \frac{n!}{j!k!l!m!} a^j b^k c^l d^m$$

- Using multipole expansion, arrive at a general expression for the perturbative Hamiltonian representing *i* multipole kicks:

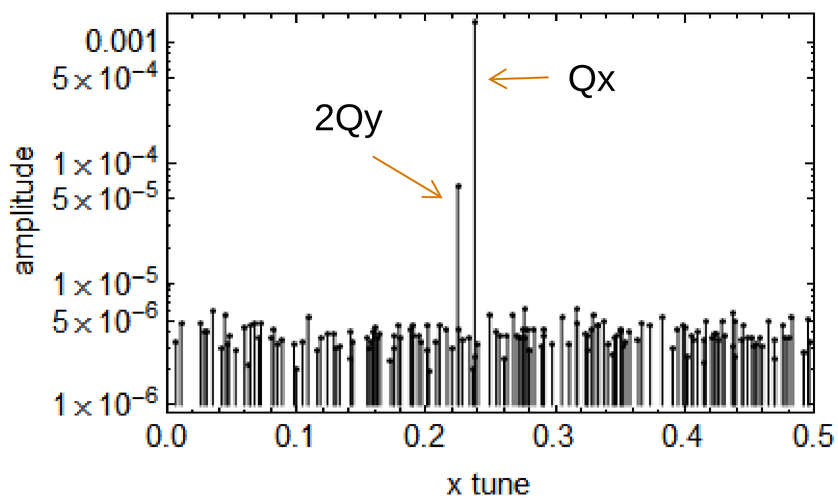
$$h = \sum_{jklm} h_{jklm} (2J_x)^{\frac{j+k}{2}} (2J_x)^{\frac{l+m}{2}} e^{i\left[(j-k)\varphi_x + (l-m)\varphi_y\right]}$$

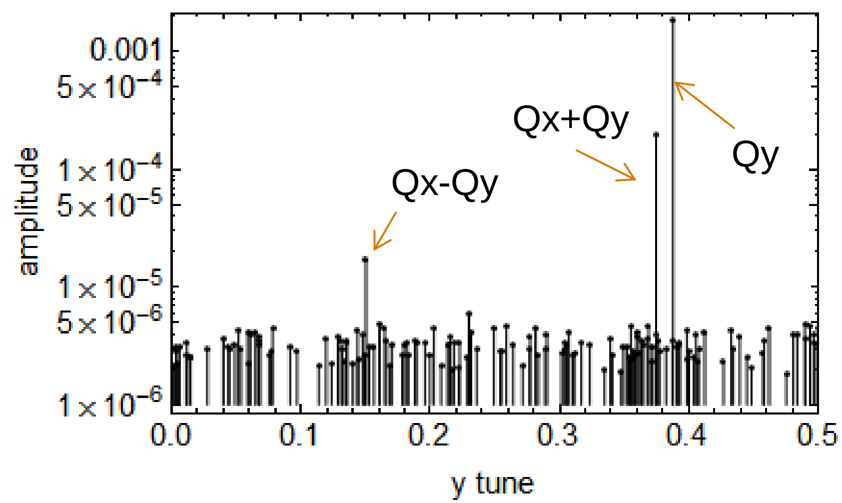
$$h_{jklm} = -\frac{q}{p_0 2^n n} \frac{n}{j!k!l!m!} \sum_i L_i \beta_{xi}^{\frac{j+k}{2}} \beta_{yi}^{\frac{l+m}{2}} V_{ni} e^{i\left[(j-k)\varphi_{xi} + (l-m)\varphi_{yi}\right]}$$

($V_{ni} = A_{ni}$ (skew coefficient) if l+m is odd;
$V_{ni} = B_{ni}$ (normal coefficient) if l+m is even)

Relation between Hamiltonian driving terms h_{jklm} and observable spectrum

- Frequencies excited by multipole perturbations are visible in the Fourier spectrum of the beam trajectory
- The Hamiltonian term hjklm excites
  - the resonance $(j-k)Q_x+(l-m)Q_y$=integer
  - the horizontal spectrum line $(1-j+k)Q_x+(m-l)Q_y$ (if $l \neq 0$)
  - the vertical spectrum line $(k-j)Q_x+(1-l+m)Q_y$ (if $l \neq 0$)
- Example: tracking simulation with normal sextupole errors; Q_x=4.238, Q_y=4.389





Summary of resonances and spectral lines excited by driving terms h_{jklm} (up to n=3)

Normal Quadrupole

Term	Res.	H line	V line
h_{0011}	(0,0)	–	(0,1)
h_{0020}	(0,2)	–	(0,–1)
h_{1100}	(0,0)	(1,0)	–
h_{2000}	(2,0)	(–1,0)	–

Skew Quadrupole

Term	Res.	H line	V line
h_{0110}	(–1,1)	–	(1,0)
h_{1001}	(1,–1)	(0,1)	–
h_{1010}	(1,1)	(0,–1)	(–1,0)

Normal Sextupole

Term	Res.	H line	V line
h_{0111}	(–1,0)	–	(1,1)
h_{0120}	(–1,2)	–	(1,–1)
h_{1002}	(1,–2)	(0,2)	–
h_{1011}	(1,0)	(0,0)	(–1,1)
h_{1020}	(1,2)	(0,–2)	(–1,–1)
h_{1200}	(–1,0)	(2,0)	–
h_{3000}	(3,0)	(–2,0)	–

Skew Sextupole

Term	Res.	H line	V line
h_{0012}	(0,–1)	–	(0,2)
h_{0030}	(0,3)	–	(0,–2)
h_{0210}	(–2,1)	–	(2,0)
h_{1101}	(0,–1)	(1,1)	–
h_{1110}	(0,1)	(1,–1)	(0,0)
h_{2001}	(2,–1)	(–1,1)	–
h_{2010}	(2,1)	(–1,–1)	(–2,0)

- Terms with l+m=even correspond to normal multipoles, l+m=odd to skew multipoles
- A single line in the spectrum can be excited by several Hamiltonian driving terms
- Theory predicts amplitudes and phase of spectral line from each driving term

Amplitude and phase of Hamiltonian driving terms h_{jklm}

	Generating Function	Spectral Line	Plane
Amplitude	$\lvert f_{jklm}\rvert$	$2\cdot j\cdot (2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}}\lvert f_{jklm}\rvert$	Horizontal
		$2\cdot l\cdot (2I_x)^{\frac{j+k}{2}}(2I_y)^{\frac{l+m-1}{2}}\lvert f_{jklm}\rvert$	Vertical
Phase	ϕ_{jklm}	$\phi_{jklm}+\psi_{x_0}-\frac{\pi}{2}$	Horizontal
		$\phi_{jklm}+\psi_{y_0}-\frac{\pi}{2}$	Vertical

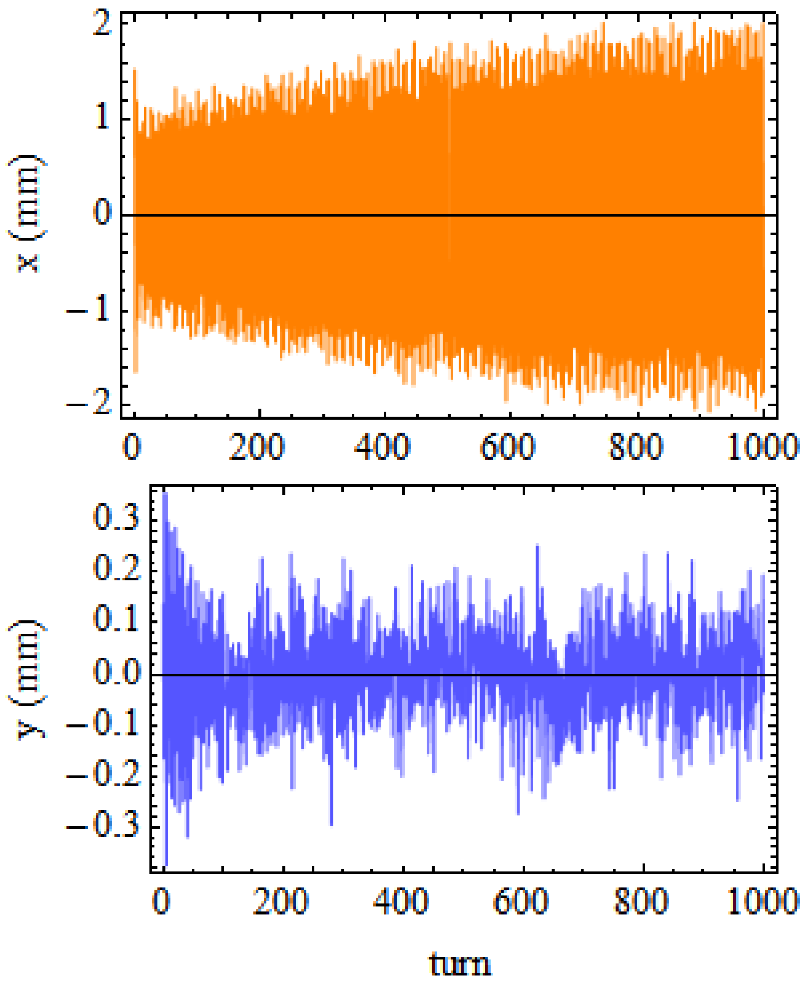
$$f_{jklm}=\frac{h_{jklm}}{1-e^{-i2\pi[(j-k)Q_x+(l-m)Q_y]}}$$

- Amplitude and phase of a resonance driving term are identified via comparison w/ amplitude and phase of spectral lines
- Once driving terms are known, can find settings for a pair of corrector magnets that will compensate for each driving term

Part III
Spectrum measurements

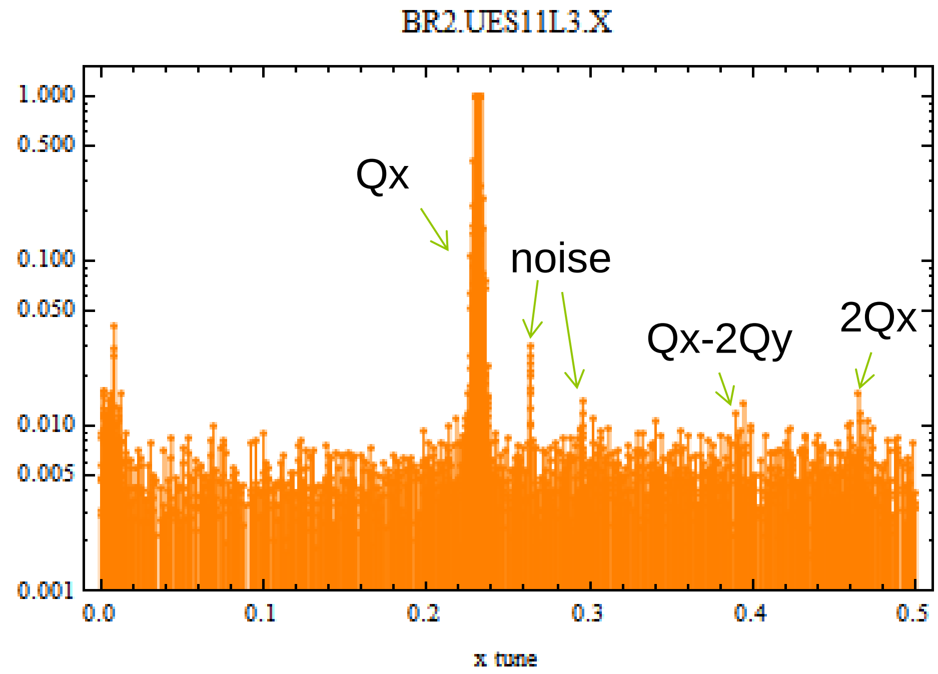
Turn-by-turn trajectory measurements

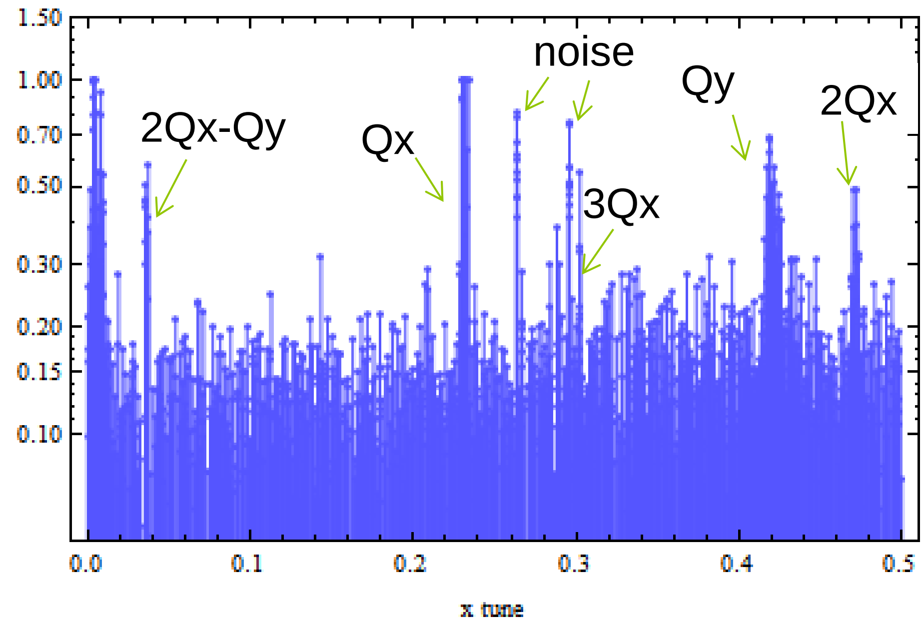
- Trial of trajectory measurements was done with three BPMs
- Tune kicker and transverse damper used to cause transverse oscillations
- Oscillation amplitude from tune kicker or damper was smaller than desired (~1 mm peak-to-peak)
- Taking advantage of transverse instability gives better horizontal oscillation amplitude



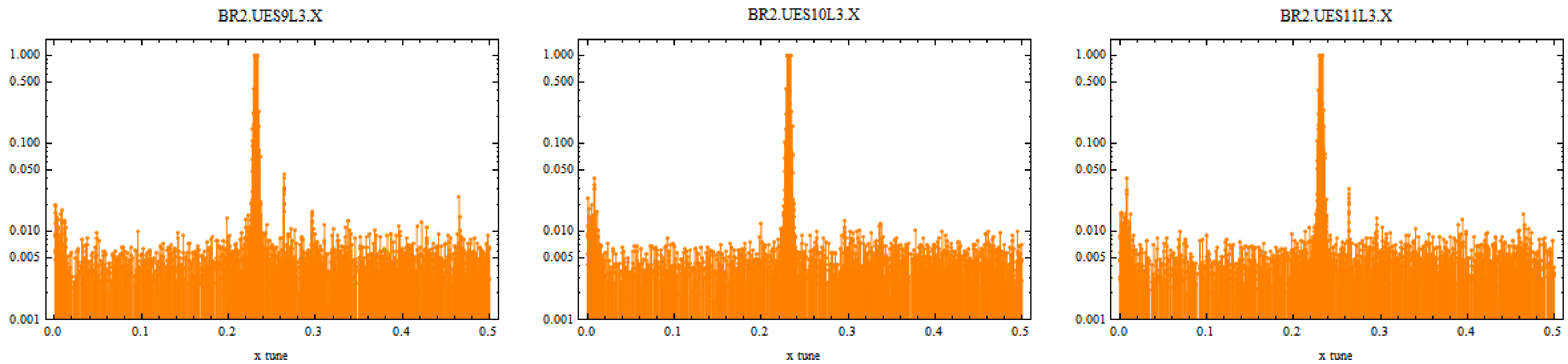
Spectra of measured trajectories

- Orange - measured x spectrum
  blue - measuredy spectrum
  red - tracking w/ normal sext errors
- Peaks visible near (but not exactly on) resonance frequencies
- Possible appearance of skew octupole term h0121:
  H line (2,-1), V line (1,0)
- Amplitude of peaks is small relative to noise floor; phase and amplitude inconsistent on repeated pulses
- Spectrum also shows noise peaks, always visible at ~263 and 297 KHz





Spectra of measured trajectories



Summary

- Hamiltonian driving terms describing nonlinear imperfections can be determined from measured beam trajectory spectra
- Driving terms can then be compensated with corrector magnets

- Trial measurements were made in PSB before LS1, but spectral analysis was complicated by several factors:
  - low oscillation amplitude/signal-to-noise ratio
  - large tune ripple
  - “noise” peaks which are always present at ~263 and 297 KHz
- Nonetheless, first measurements show some hints of higher-order frequency components
- After LS1, measurements will be repeated with abovementioned problems (hopefully) resolved, and we’ll try to compensate whatever resonance driving terms we observe

Thank you for your attention.